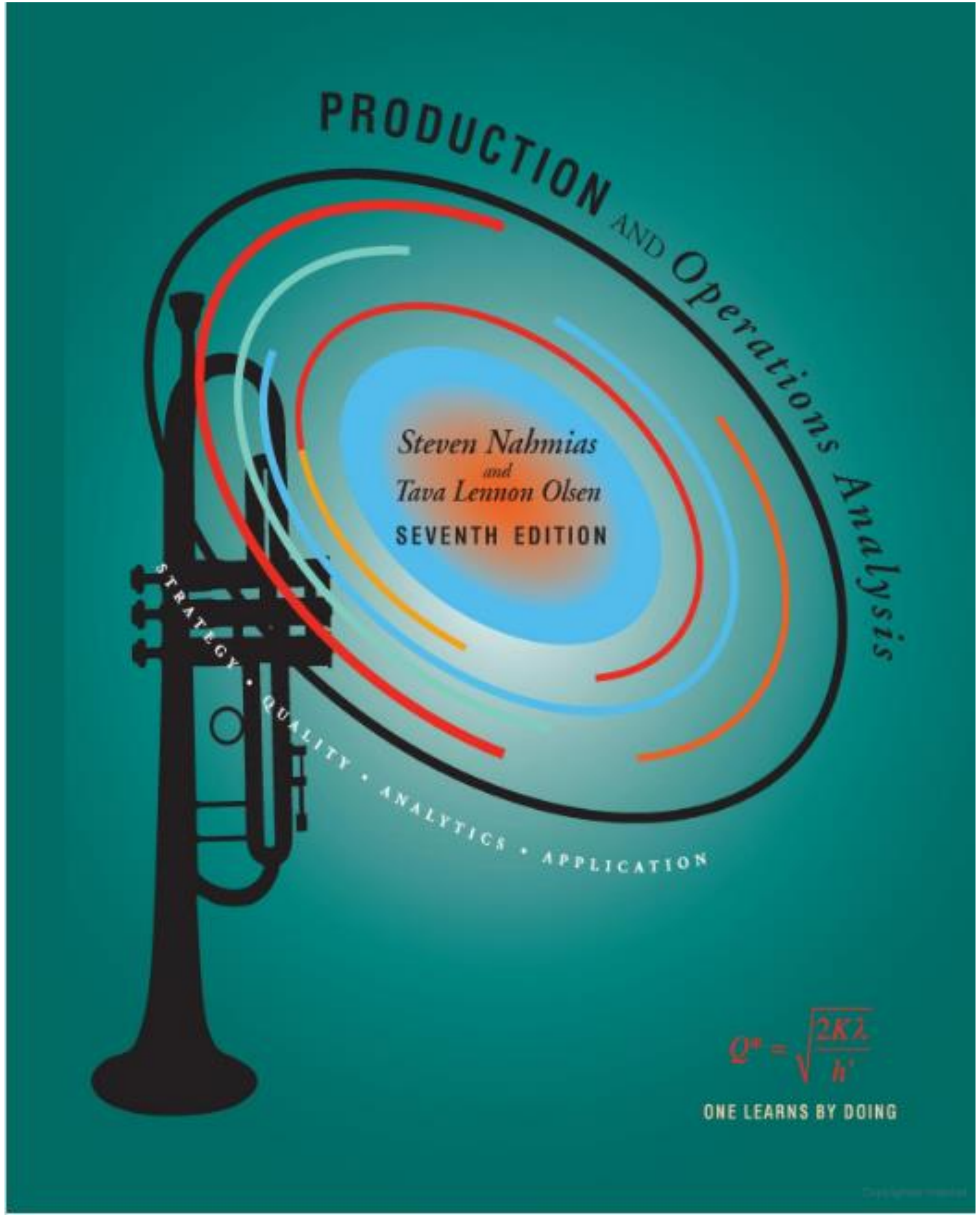
FACULTY OF ECONOMICS AND BUSINESS ADMINISTRATION

DEPARTMENT OF BUSINESS INFORMATICS AND OPERATIONS MANAGEMENT
RESEARCH GROUP OPERATIONS AND PRODUCTION MANAGEMENT

OPERATIONS MANAGEMENT (F000242)

Prof. dr. Veronique Limère


PRODUCTION AND Operations Analysis
Steven Nahmias
and
Tava Lennon Olsen
SEVENTH EDITION
STRATEGY • QUALITY • ANALYTICS • APPLICATION
$Q^* = \sqrt{\frac{2K\lambda}{h'}}$
ONE LEARNS BY DOING

Introduction

Inventory Management

Forecasting

Manufacturing Planning & Control

Processes

DEPARTMENT OF BUSINESS INFORMATICS AND OPERATIONS MANAGEMENT
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You are welcome to be seated on the first rows!

FORECASTING

Prof. dr. Veronique Limère

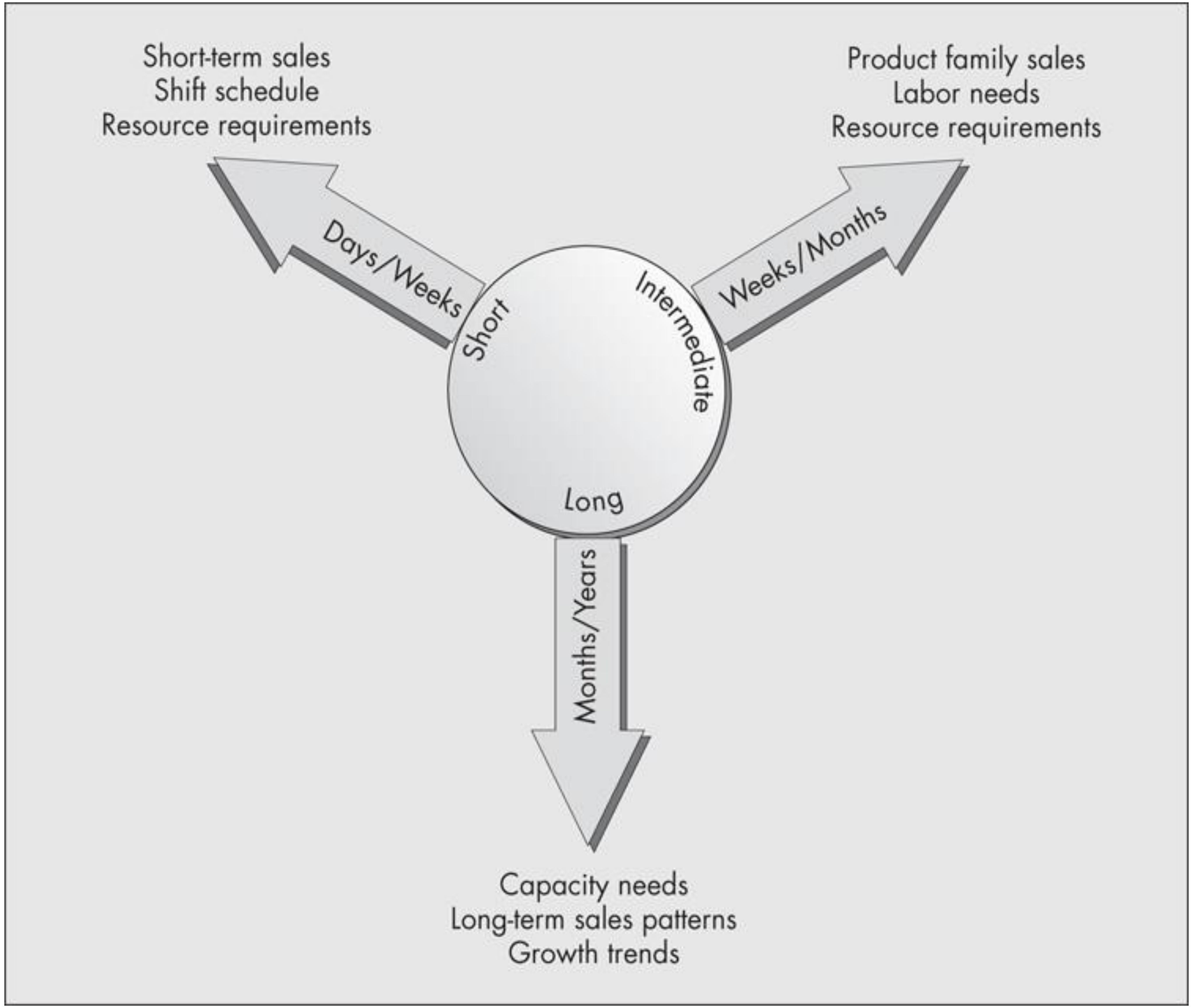
INTRODUCTION TO FORECASTING

- What is forecasting?

  Primary function is to predict the future
- Why are we interested?

  Affects the decisions we make today
- Examples: who uses forecasting in their jobs?
  - Forecast demand of products and services
    - To plan capacity
    - To plan manpower
    - To plan inventory and material needs daily
  - Forecast resource availability

FORECAST HORIZONS IN OPERATIONS MANAGEMENT



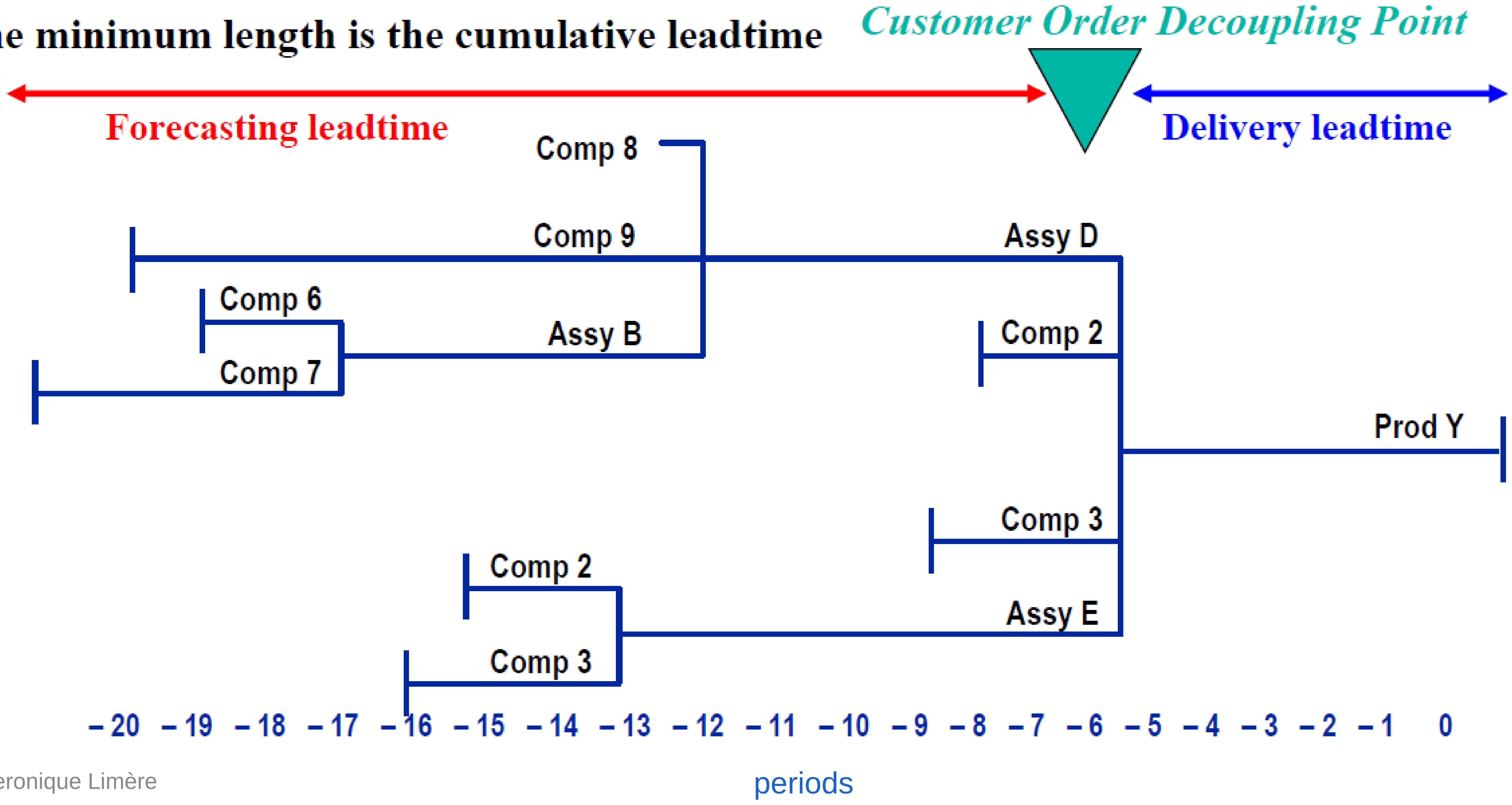
WHAT MAKES A GOOD FORECAST

- It should be timely
- It should be as accurate as possible
- It should be reliable
- It should be in meaningful units
- It should be presented in writing
- The method should be easy to use and understand in most cases

FORECAST HORIZON

How far in the future must our 'Demand Forecasting look?

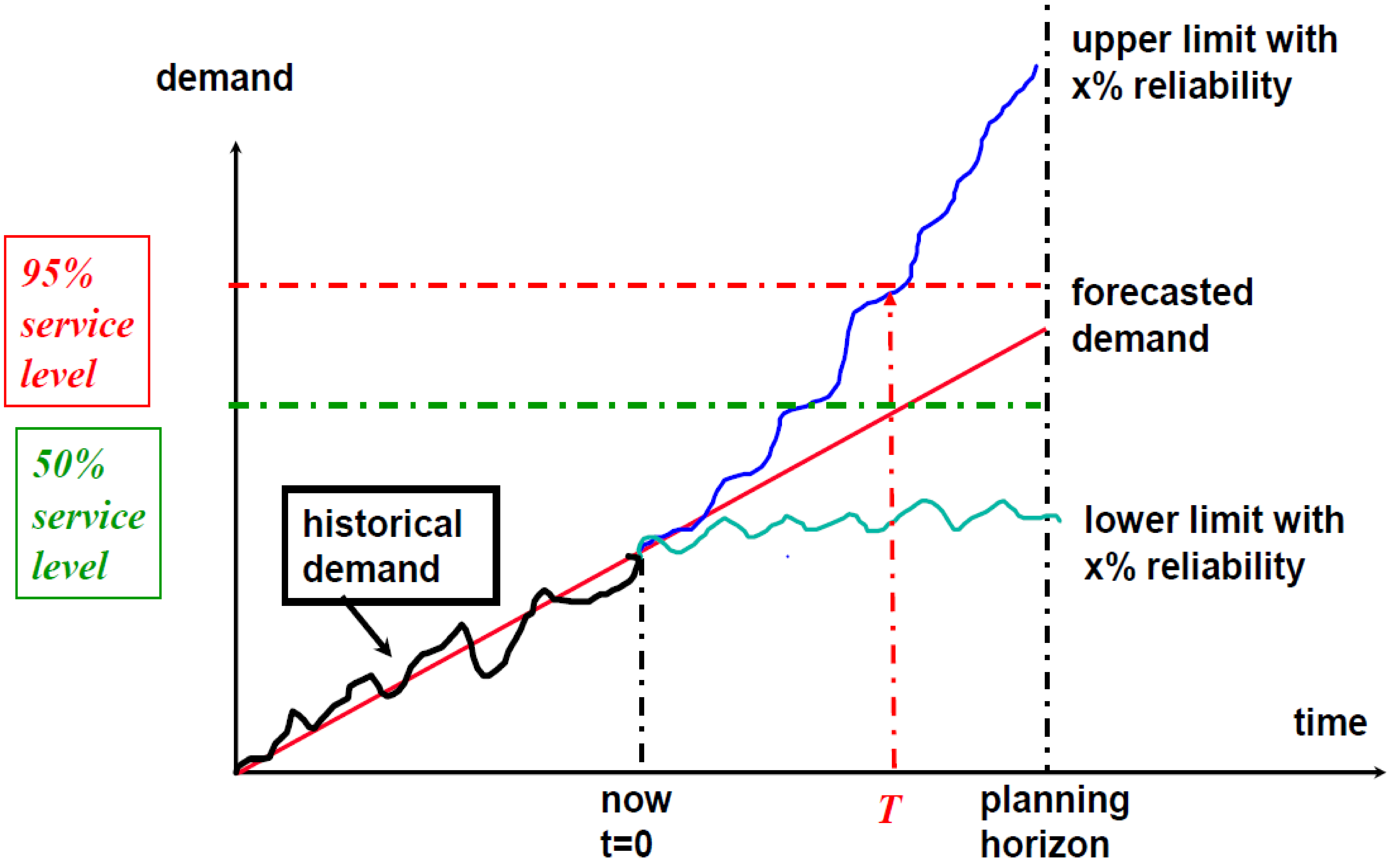
The minimum length is the cumulative leadtime



HOW MUCH INVENTORY DO I NEED AT TIME T?

If a forecast is used to estimate the mean demand, keep safety stocks in order to protect against the error in this forecast

Use σ_e instead of σ



OUTLINE

- Introduction
- Subjective versus objective forecasting methods
- Evaluation of forecasts
- Forecasting for stationary series
- Trend-based methods
- Methods for seasonal series
- Conclusion

SUBJECTIVE FORECASTING METHODS

- Sales force composites

  Aggregation of sales personnel estimates
- Customer surveys
- Jury of executive opinion
- The Delphi method

  Individual opinions are compiled and reconsidered. Repeat until an overall group consensus is (hopefully) reached.

OBJECTIVE FORECASTING METHODS

Two primary methods: *causal models* and *time series methods*

1. Causal models

Let Y be the quantity to be forecasted and $(X_1, X_2, \ldots, X_n)$ are n variables that have predictive power for Y.

A causal model is $Y = f(X_1, X_2, \ldots, X_n)$.

A typical relationship is a linear one:

$$Y = a_0 + a_1X_1 + \ldots + a_n X_n$$

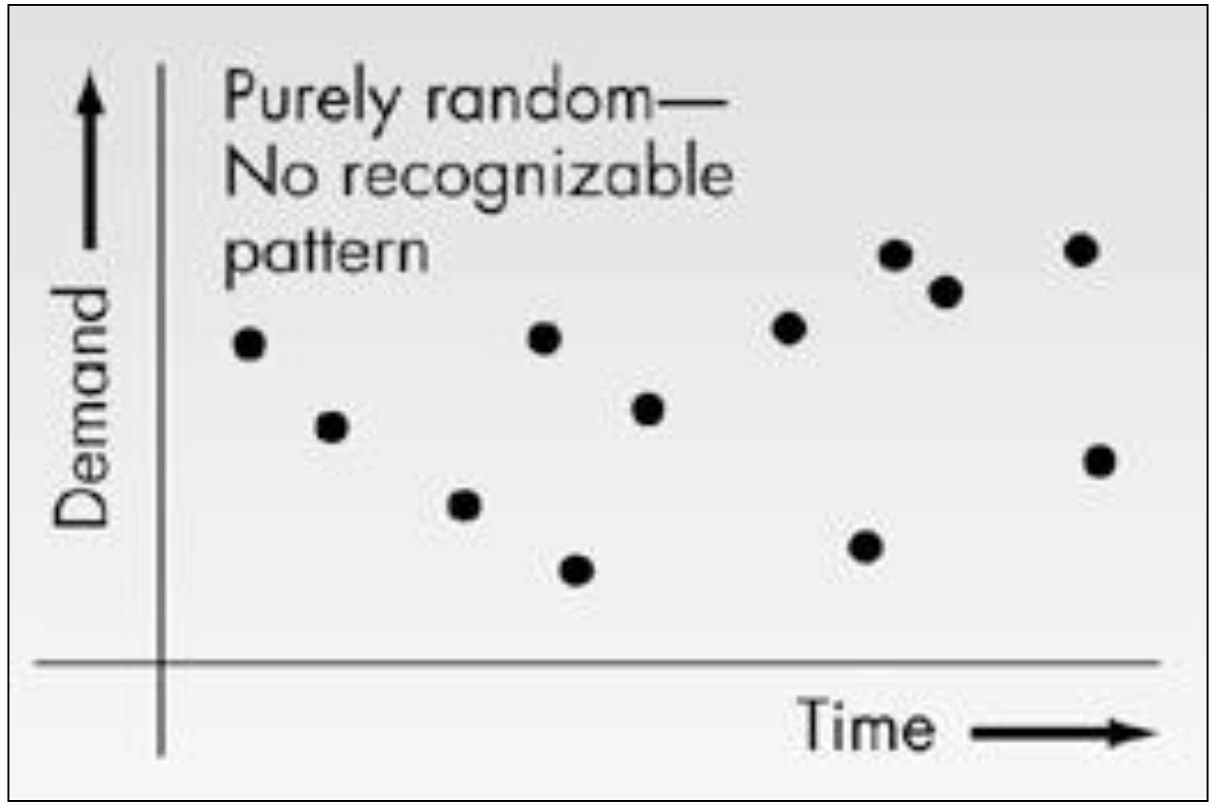
OBJECTIVE FORECASTING METHODS

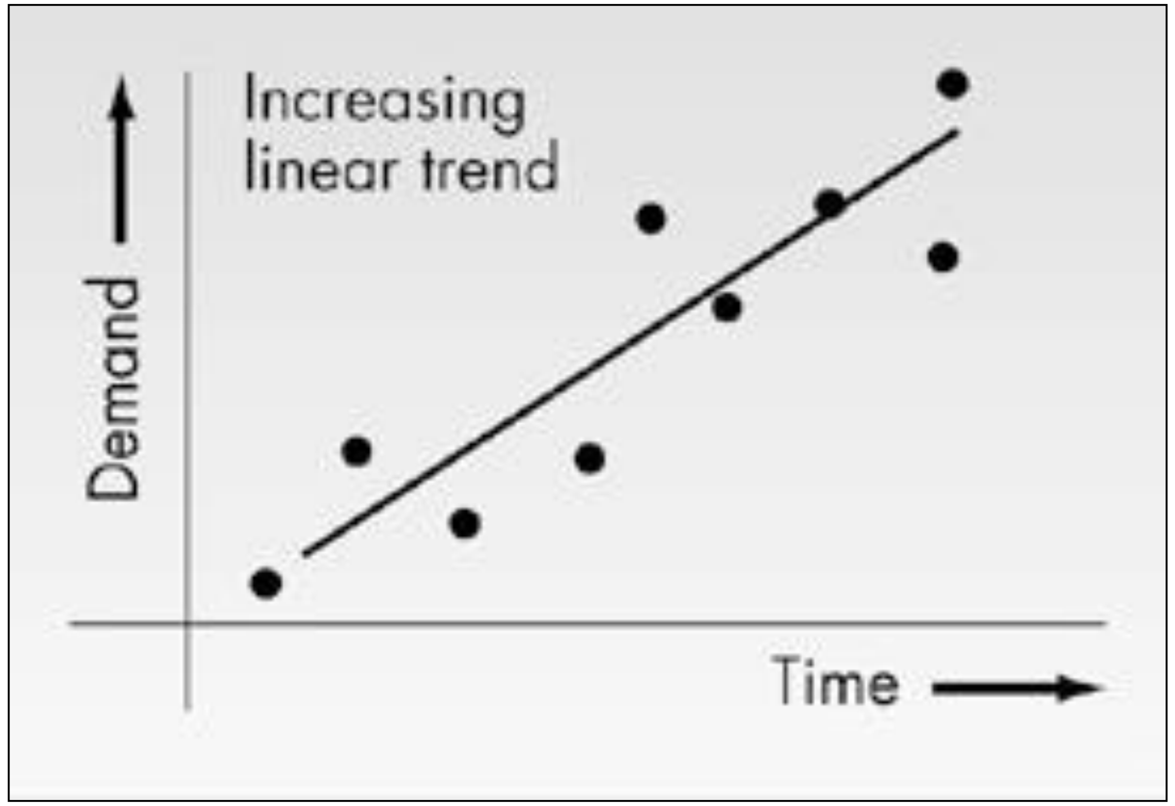
Two primary methods: *causal models* and *time series methods*

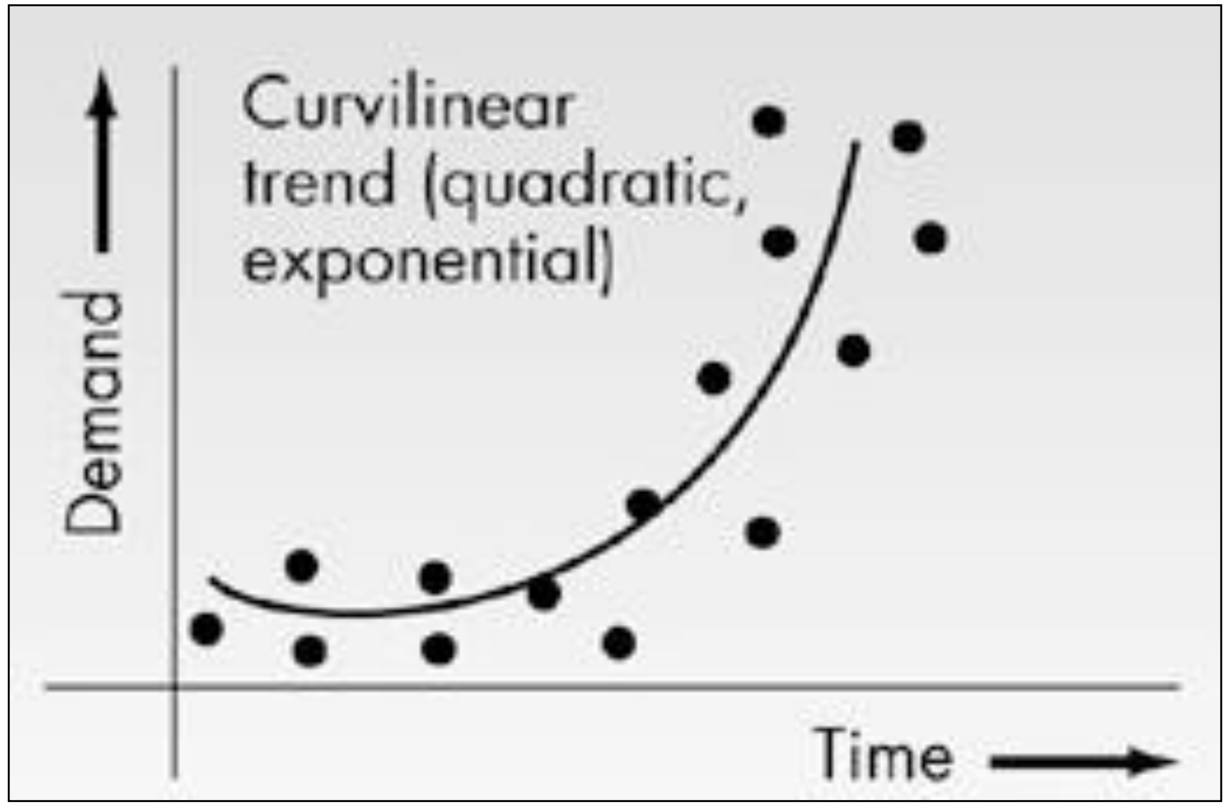
2. Time series methods

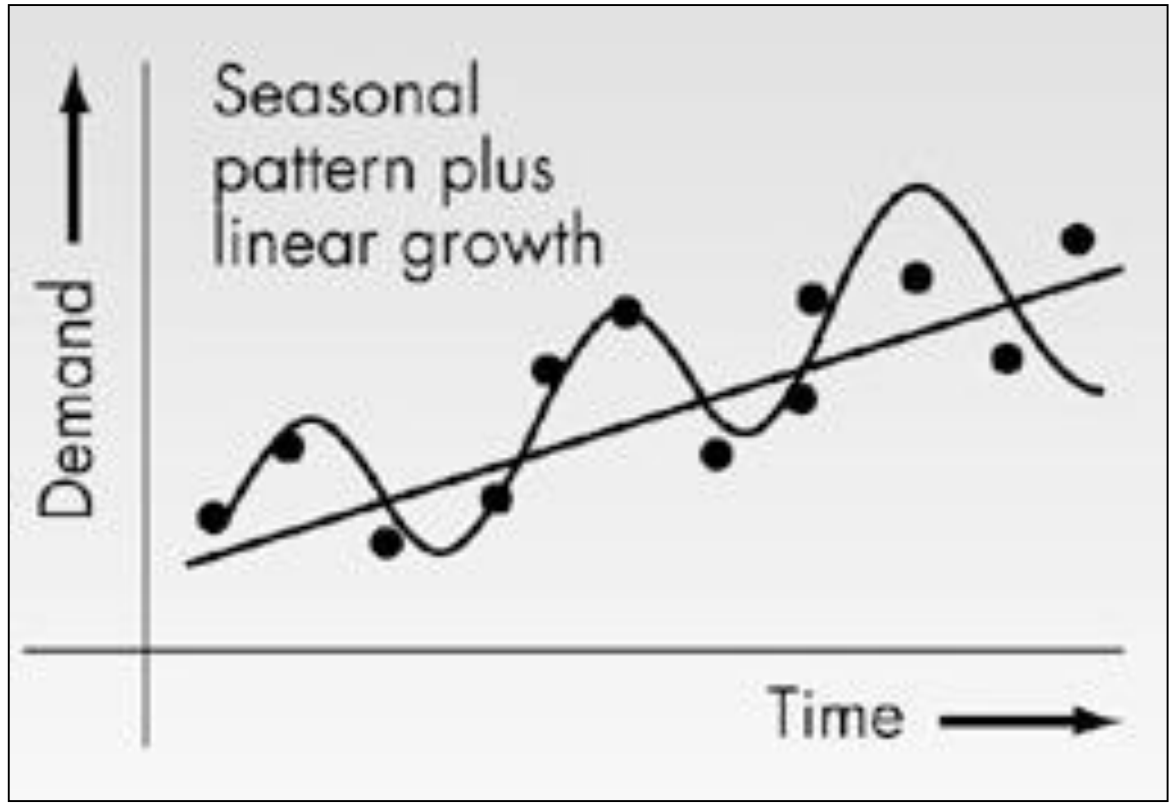
- A collection of past values of the variable being predicted
- Also known as naïve methods
- Goal is to isolate patterns in past data
  - Trend
  - Seasonality
  - Cycles
  - Randomness

PATTERNS IN PAST DATA









NOTATION CONVENTIONS FOR TIME SERIES METHODS

- $D_1, D_2, \ldots, D_t, \ldots$ = past values of the series to be predicted (demand)
- If we are making a forecast in period t, assume we have observed D_t , D_{t-1} etc.
- $F_{t,\, t+\tau}$ = forecast made in period t for the demand in period $t + \tau$,

  where $\tau = 1, 2, 3, \ldots$
- For one-step-ahead forecasts, use shorthand notation $F_t = F_{t-1,\, t}$
- A time series forecast is obtained by applying some set of weights $a_1, a_2, \ldots$ to past data:

$$F_t = \sum_{n=1}^{\infty} a_n D_{t-n}$$

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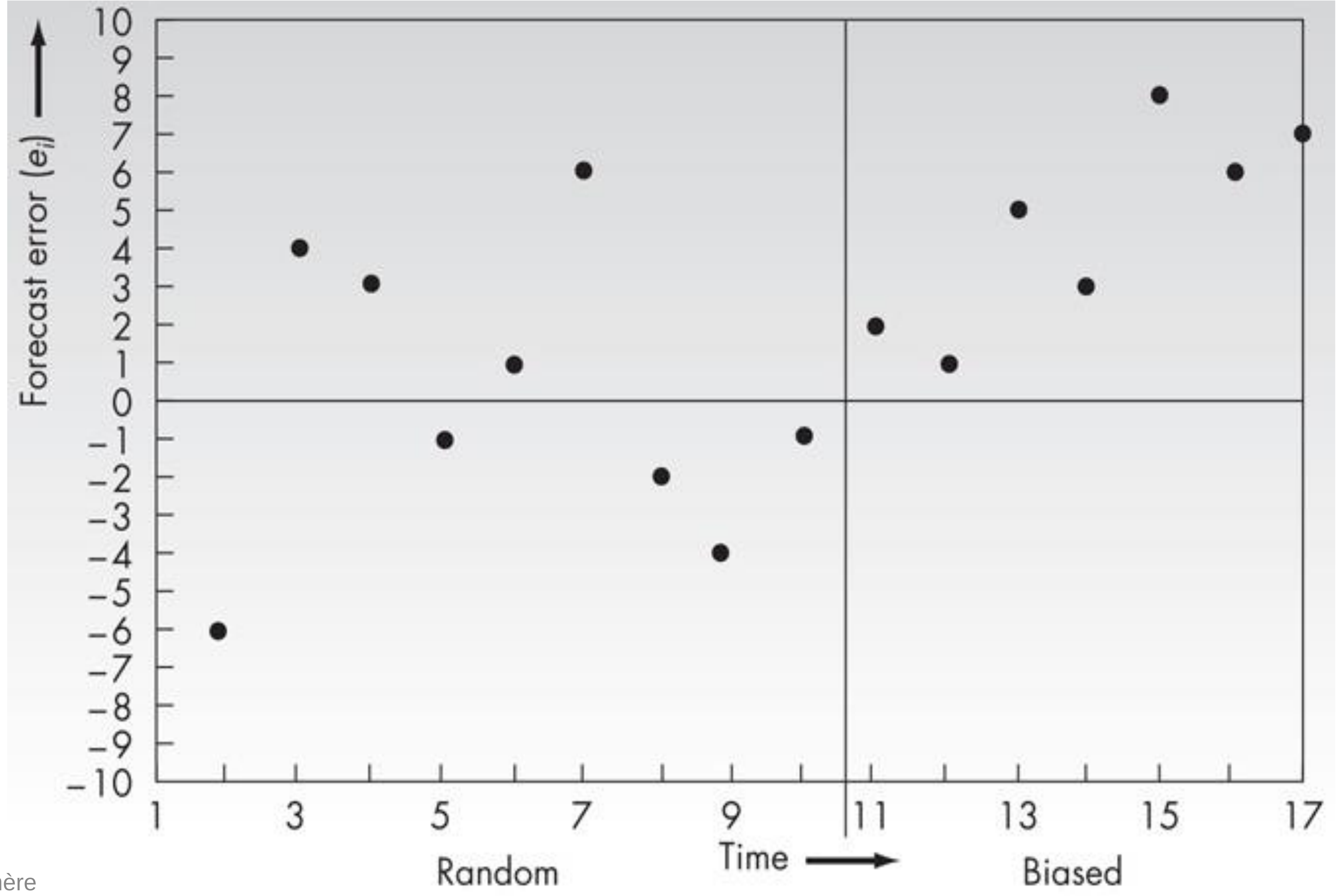
EVALUATION OF FORECASTS

- The forecast error in period t, e_t, is the difference between the forecast for demand in period t and the actual value of demand in t.
  - For a multiple-step-ahead forecast: $e_t = F_{t-\tau, t} - D_t$.
  - For one-step-ahead forecast: $e_t = F_t – D_t$

1. Forecasts should be **unbiased**: $E(e_i) = 0$
2. Different measures of **forecast accuracy**

FORECASTS ERRORS OVER TIME TO DETECT BIAS



MEASURES OF FORECAST ACCURACY

- Two common measures:
  - **Mean absolute deviation**

$$MAD = (1/n)\sum_{i=1}^{n}|e_i|$$

    - This measure is often preferred (no squaring)
    - When forecasts errors are normally distributed (as generally assumed): $\sigma_e \approx 1.25 \times MAD$
  - **Mean squared error**

$$MSE = (1/n)\sum_{i=1}^{n} e_i^2$$

    - Similar to the variance of a random sample

MEASURES OF FORECAST ACCURACY

- Other measures are used as well, e.g.:
  - **Mean absolute percentage error**

$$MAPE = \left[(1/n) \sum_{i=1}^{n} |e_i / D_i| \right] \times 100$$

Not dependent on the magnitude of the values

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FORECASTING FOR STATIONARY SERIES

- A stationary time series has the form:

$$D_t = \mu + \varepsilon_t$$

where μ is an unknown constant and ε_t is a random variable with mean 0 and variance σ^2

  - Stationarity means no growth or decline in the series and variation relatively constant
  - Stationarity does not imply independence: it is possible that D_i and D_j are dependent random variables

- Two common methods for forecasting stationary series are moving averages and exponential smoothing.

MOVING AVERAGES

- Simple moving averages
- $MA(N)$ uses the mean of the N most recent observations as the forecast
- For a one-step-ahead forecast:

$$F_t = (1/N)(D_{t-1} + D_{t-2} + \cdots + D_{t-N})$$

$$F_{t+1} = (1/N) \sum_{i=t-N+1}^{t} D_i = F_t + (1/N)[D_t - D_{t-N}]$$

- Multiple-step-ahead and one-step-ahead forecasts are identical

MOVING AVERAGE - EXAMPLE

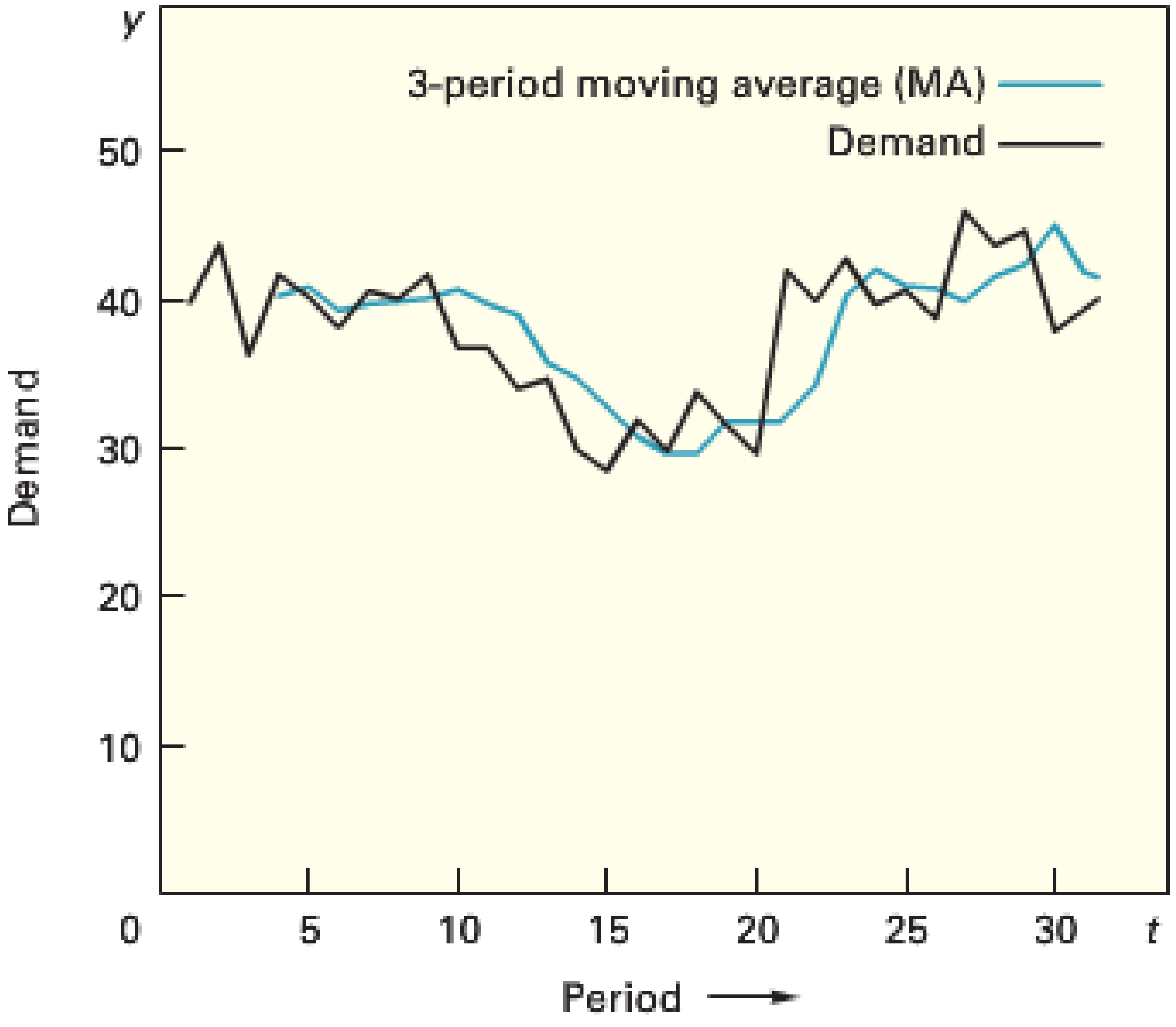
MONTH	Demand	Month	Demand
January	89	July	223
February	57	August	286
March	144	September	212
April	221	October	275
May	177	November	188
June	280	December	312

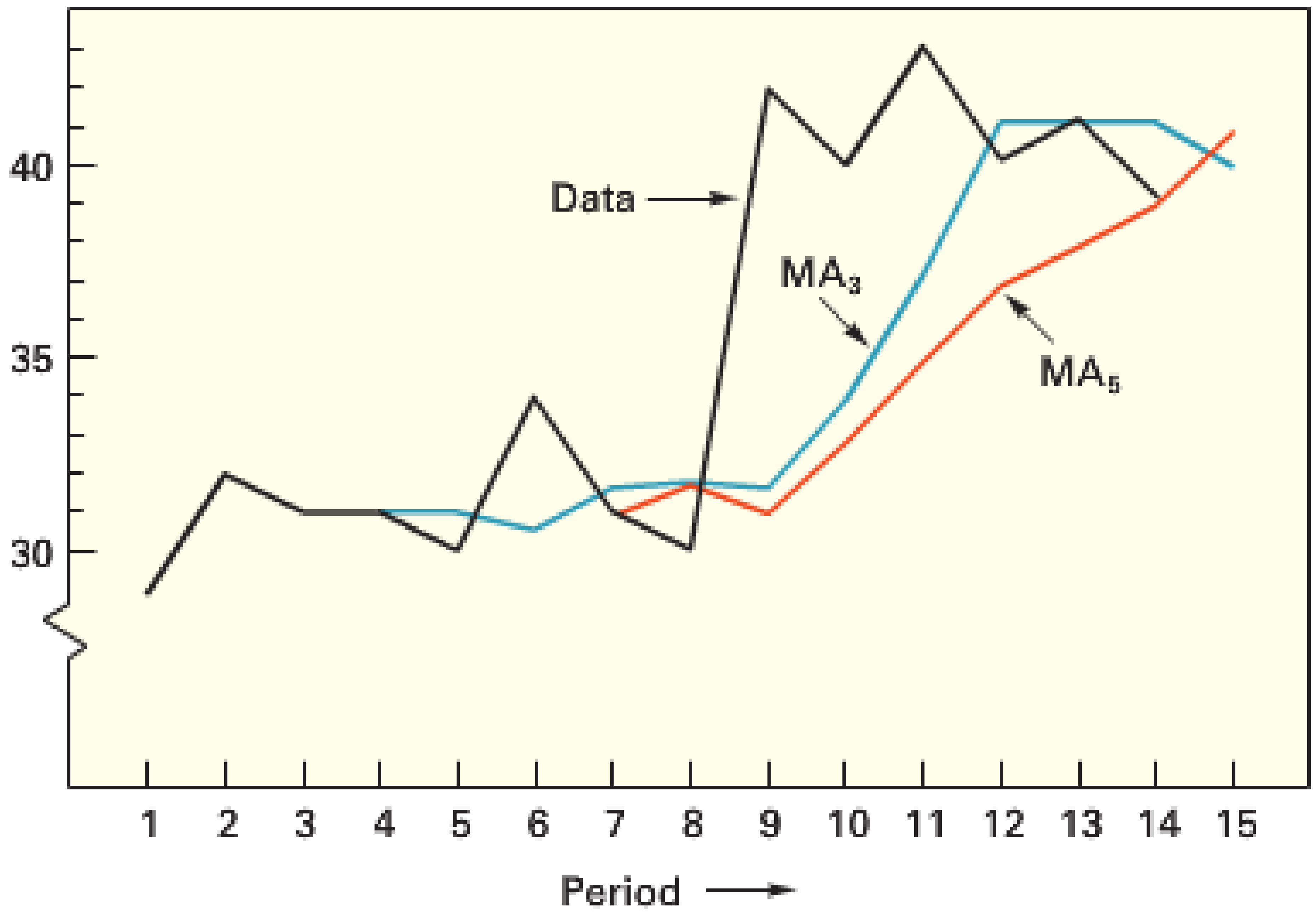
3 month MA: (oct+nov+dec)/3=258.33

6 month MA: (jul+aug+…+dec)/6=249.33

12 month MA: (Jan+feb+…+dec)/12=205.33

MOVING AVERAGE LAGS BEHIND A TREND





SUMMARY OF MOVING AVERAGES

- *Advantages of Moving Average method*
  - ➢ Easily understood
  - ➢ Easily computed
  - ➢ Provides stable forecasts:

- *Disadvantages of Moving Average method*
  - ➢ Requires saving lots of past data points: at least the N periods used in the moving average computation
  - ➢ Lags behind a trend
  - ➢ Ignores complex relationships in data

WHAT ABOUT WEIGHTED MOVING AVERAGES?

- This method looks at past data and tries to logically attach importance to certain data over other data
- Weighting factors must add to one: Why?
- Can weigh recent higher than older, or specific data above others
  - ➢ If forecasting staffing, we could use data from the last four weeks where Tuesdays are to be forecast.
  - ➢ Weighting on Tuesdays is: T_{-1} is .25; T_{-2} is .20; T_{-3} is .15; T_{-4} is .10 and Average of all other days is weighed .30.

EXPONENTIAL SMOOTHING

$$F_{t+1} = \alpha D_t + (1 - \alpha) F_t$$

where $0 < \alpha \leq 1$ is the smoothing constant

– Also: $F_{t+1} = F_t - \alpha (F_t - D_t) = F_t - \alpha e_t$

  Smoothing:
  - if F_t is too high, e_t is positive, and the adjustment is to decrease the forecast
  - if F_t is too low, e_t is negative, and the adjustment is to increase the forecast

– A type of weighted moving average that applies declining weights to past data

  What are the weights?

EXPONENTIAL SMOOTHING

$F_{t+1} = \alpha D_t + (1 - \alpha) F_t$

$= \alpha D_t + (1 - \alpha)(\alpha D_{t-1} + (1 - \alpha) F_{t-1})$

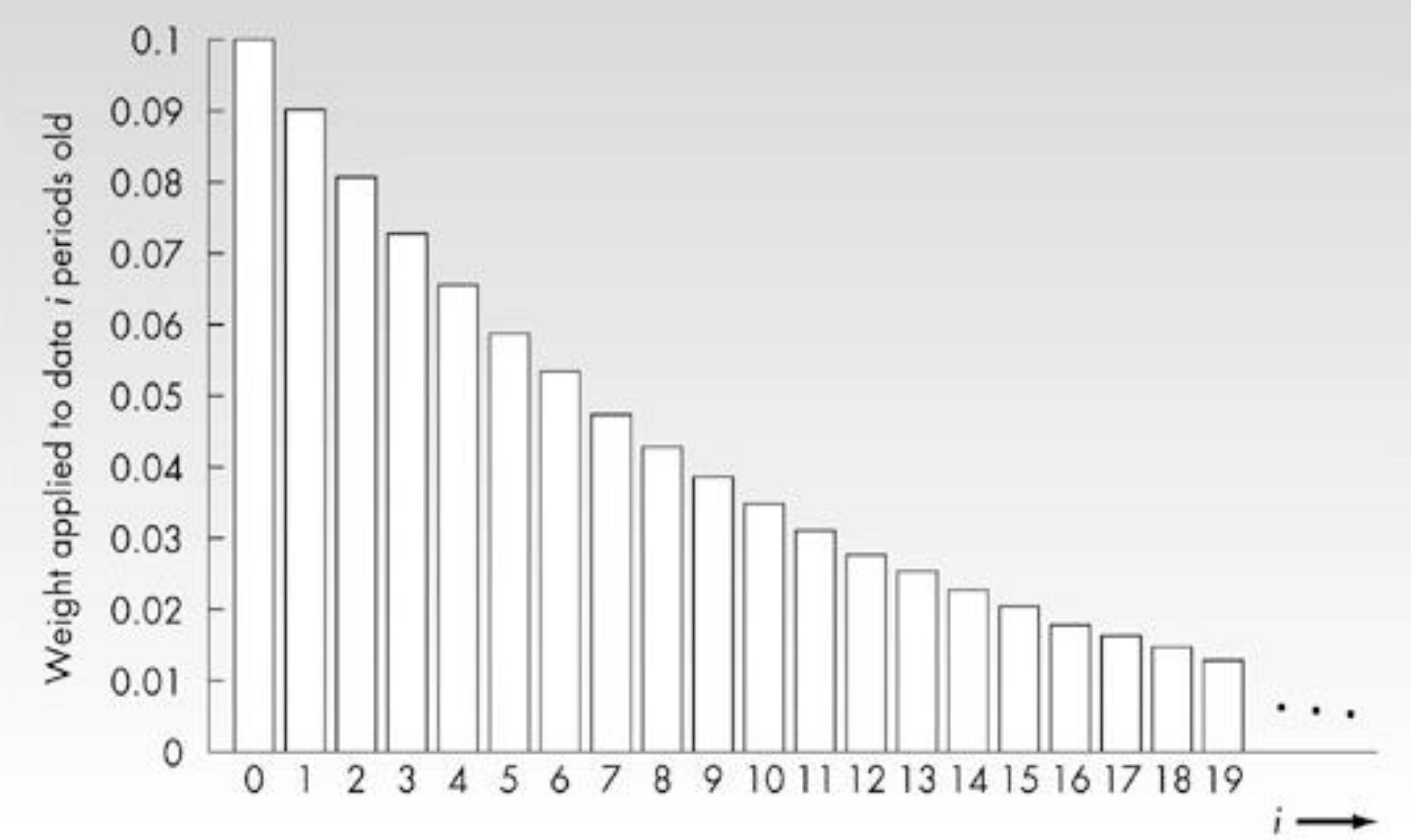
Infinite expansion for F_{t+1}:

$$F_{t+1} = \alpha D_t + (1-\alpha)(\alpha) D_{t-1} + (1-\alpha)^2(\alpha) D_{t-2} + \cdots$$

$$F_{t+1} = \sum_{i=0}^{\infty} \alpha(1-\alpha)^i D_{t-i}$$

- A set of exponentially declining weights applied to past data
- It is easy to show that the sum of the weights $\sum_{i=0}^{\infty} \alpha(1-\alpha)^i = 1$

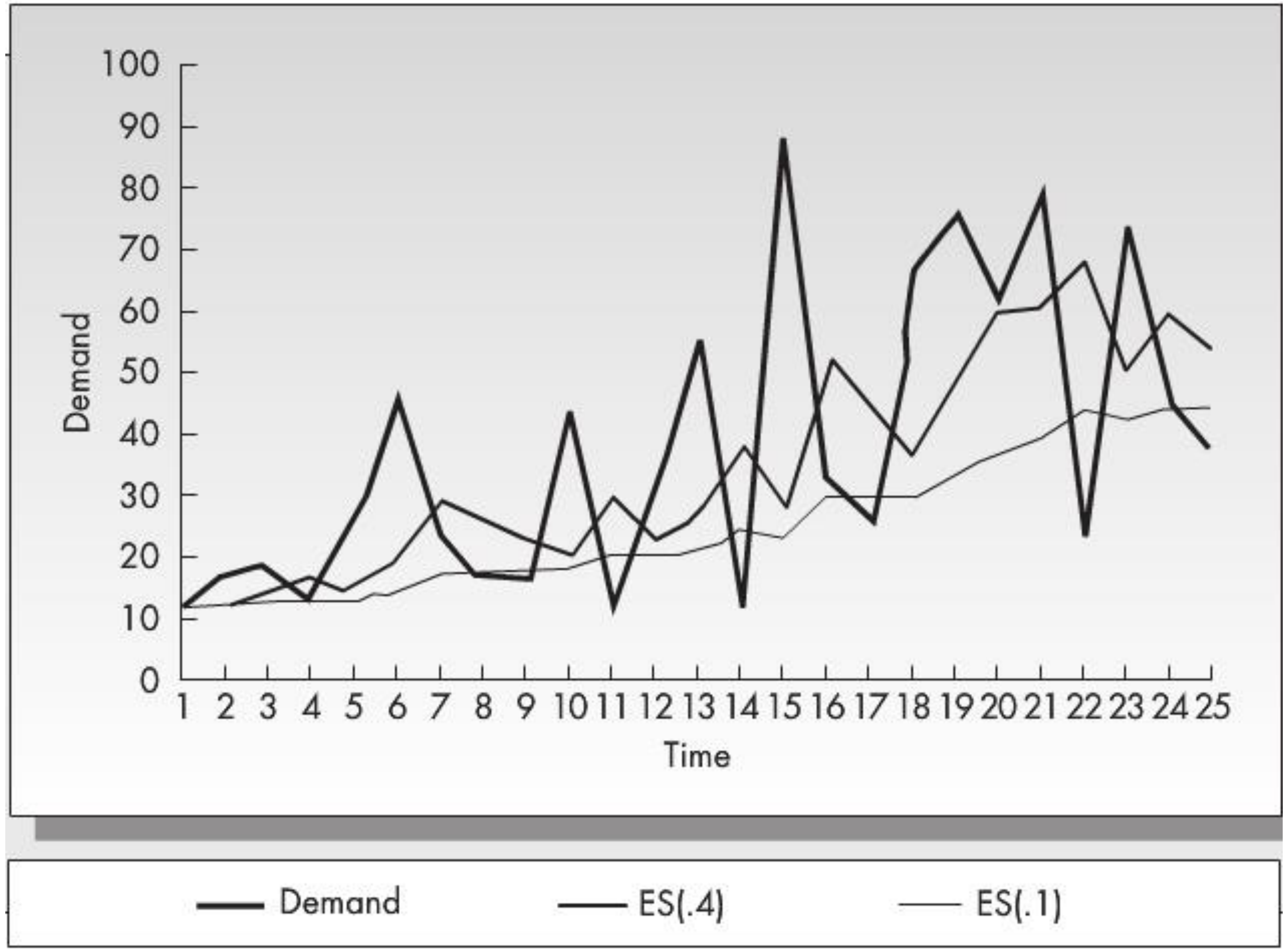
WEIGHTS IN EXPONENTIAL SMOOTHING



EFFECT OF α VALUE ON THE FORECAST

- Small values of α means that the forecasted value will be stable (show low variability)
  - Low α increases the lag of the forecast to the actual data if a trend is present
- Large values of α mean that the forecast will more closely track the actual time series (quick reaction to changes)
- For production applications, stable demand forecasts are desired
  - Therefore, a small α is recommended around 0.1 to 0.2

EFFECT OF α VALUE ON THE FORECAST



AN EXAMPLE

- Given sales history of: Jan 23.3 Feb 72.3 Mar 30.3 Apr 15.5
- The January forecast was 25
- Using α = .15

- Forecast for Feb: $\alpha D_{jan} + (1- \alpha)F_{jan}$ = .15*23.3 + (.85)*25 = 24.745
- Forecast for Mar: $\alpha D_{feb} + (1- \alpha)F_{feb}$ = .15*72.3 + (.85)*24.745 = 31.88
- Forecast for Apr: $\alpha D_{mar} + (1- \alpha)F_{mar}$ = .15*30.3 + .85*31.88 = 31.64
- Forecast for May: $\alpha D_{apr} + (1- \alpha)F_{apr}$ = .15*15.5 + .85*31.64 = 29.22

COMPARISON OF MA AND ES

Similarities

- Both methods are appropriate for stationary series
- Both methods lag behind a trend
- Both methods depend on a single parameter
- For both methods multiple-step-ahead and one-step-ahead forecasts are identical

COMPARISON OF MA AND ES

Similarities

- Both methods are unbiased
- One can achieve the same distribution of forecast error by equating the average age of data for the two methods:

$$(1/N)(1 + 2 + 3 + \cdots + N) = \sum_{i=1}^{\infty} i\alpha(1-\alpha)^{i-1}$$

$\rightarrow \alpha = 2/(N + 1)$ or $N = (2 - \alpha)/\alpha$

- E.g., $N = 19$ for $\alpha = 0.1$, or $\alpha = 0.5$ for $N = 3$
- This will lead to roughly the same level of accuracy (but not the same forecasts)

COMPARISON OF MA AND ES

Differences

- ES carries all past history (forever!)
- MA eliminates “bad” data after N periods
- MA requires all N past data points to compute new forecast estimate while ES only requires last forecast and last observation of ‘demand’ to continue

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REGRESSION FOR TIMES SERIES FORECASTING

- Regression methods can be used when a trend is present

  Model: $\widehat{D}_t = a + bt$ (note: we only consider linear trends here)

- The least squares estimates for *a* and *b* can be computed as follows:

$$S_{xy} = n\sum_{i=1}^{n} iD_i - \left({(n*(n+1))}/{2} \right) * \sum_{i=1}^{n} D_i$$

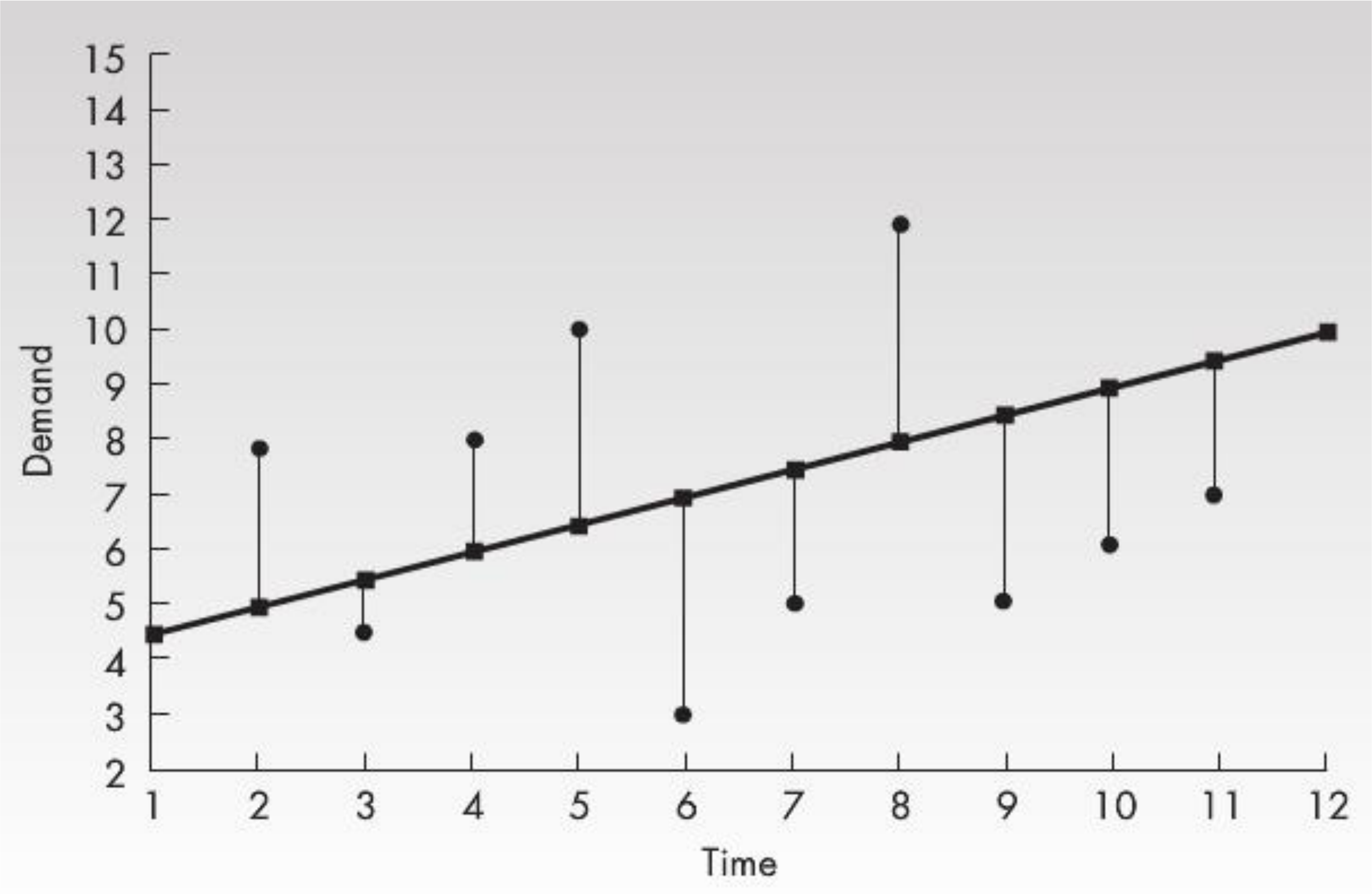
$$S_{xx} = \left({(n^2*(n+1)*(2n+1))}/{6} \right) - \left({(n^2*(n+1)^2)}/{4} \right)$$

$$a = \overline{D} - b * \left({(n+1)}/{2} \right)$$

$$b = {S_{xy}}/{S_{xx}}$$

(n is the number of observations we have)

REGRESSION FOR TIMES SERIES FORECASTING



AN EXAMPLE:

Month	# Visitors	Month	# Visitors
Jan	133	Apr	640
Feb	183	May	1875
Mar	285	Jun	2550

S_{xy} = 6*(1*133+2*183+3*285+4*640+5*1875+6*2550) –
(6*7/2)(133+183+285+640+1875+2550)] = 52548

S_{xx} = [(36*7*13)/6]-[(36*49)/4)]= 105

b = (52548/105)= 500.46

a = 944.33 – 500.46*(6+1)/2 = -807.3

AN EXAMPLE:

- Forecast for July? $a+b*7 = -807.3 + 500.46*7 = 2696$
- Forecast for August? $-807.3 + 500.46*8 = 3196$
  and continued …

- However, once we get real data for July and August, we would need to re-compute S_{xx}, S_{xy}, a and b to continue forecasting – if we wish to be accurate!

DOUBLE EXPONENTIAL SMOOTHING - HOLT

- Double exponential smoothing, using Holt's method
- To forecast when there is a linear trend present in the data
- Two smoothing constants α and β
- Separate smoothing equations:

$S_t = \alpha D_t + (1-\alpha)(S_{t-1} + G_{t-1})$ for the value of the series (the intercept)

$G_t = \beta(S_t - S_{t-1}) + (1- \beta)G_{t-1}$ for the trend (the slope)

D_t is observed demand; S_t is current estimate of intercept; G_t is current estimate of slope; S_{t-1} is last estimate of intercept; G_{t-1} is last estimate of slope

- $F_{t,t+\tau} = S_t + \tau * G_t$ τ-step-ahead forecast

DOUBLE EXPONENTIAL SMOOTHING - HOLT

- We begin with an estimate of the intercept and slope at the start (e.g., by using linear regression)
- Easier to calculate new forecasts by redefining the smoothing equations than regression analysis
- The smoothing constants may be the same, but often more stability is given to the slope estimate $(\beta \leq \alpha)$

AN EXAMPLE

- Aircraft engine failure data: 200, 250, 175, 186, 225, 285, 305, 190
- Assume $\alpha = 0.1$ and $\beta = 0.1$
- In order to get the method started: $S_0 = 200$ and $G_0 = 10$

$$S_1 = (0.1)(200) + (0.9)(200 + 10) = 209.0$$
$$G_1 = (0.1)(209 - 200) + (0.9)(10) = 9.9$$
$$S_2 = (0.1)(250) + (0.9)(209 + 9.9) = 222.0$$
$$G_2 = (0.1)(222 - 209) + (0.9)(9.9) = 10.2$$
$$S_3 = (0.1)(175) + (0.9)(222 + 10.2) = 226.5$$
$$G_3 = (0.1)(226.5 - 222) + (0.9)(10.2) = 9.6$$

and so on

AN EXAMPLE (CONTD)

- Results (one-step-ahead forecasts)

Period	Actual	Forecast	|error|
4	186	236.1	50.1
5	225	240.3	15.3
6	285	247.7	37.3
7	305	260.8	44.2
8	190	275.0	85.0

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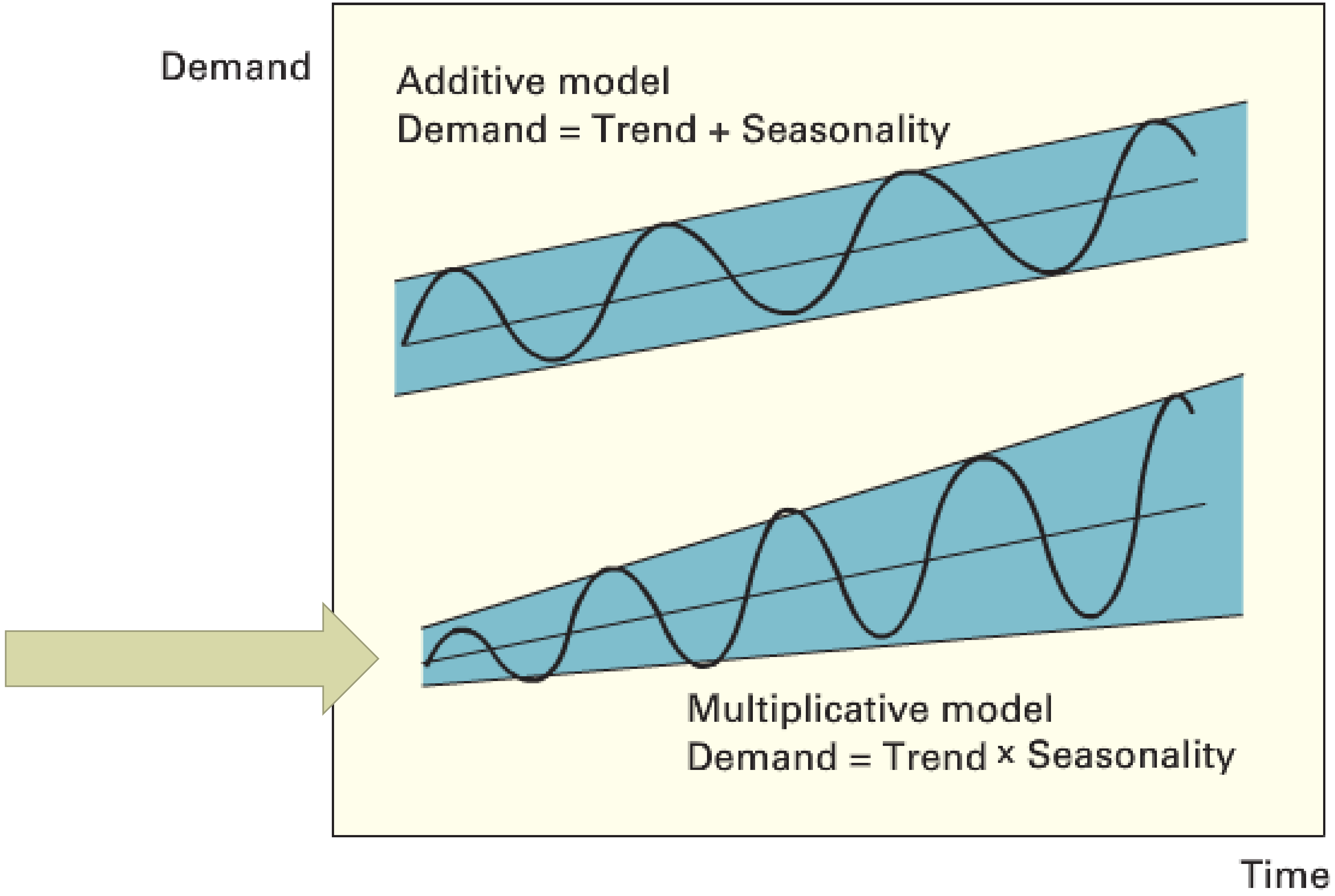
FORECASTING FOR SEASONAL SERIES

- Assumption that the underlying series has a form similar to a multiplicative model



FORECASTING FOR SEASONAL SERIES

- Seasonality corresponds to a pattern in the data that repeats at regular intervals.
- Multiplicative seasonal factors: c_t (for $1 \leq t \leq N$) where t=1 is first season of the cycle, t=2 is second season of the cycle, etc.

$$\Sigma\, c_t = N$$

- ➢ c_t = 1.25 implies a ‘demand’ 25% higher than the baseline
- ➢ c_t = 0.75 implies 25% lower than the baseline

FORECASTING FOR SEASONAL SERIES

Using seasonal relatives

1. *Deseasonalize data*

   Done in order to get a clearer picture of the nonseasonal (e.g., trend) components of the data series

   Divide each data point by its seasonal relative

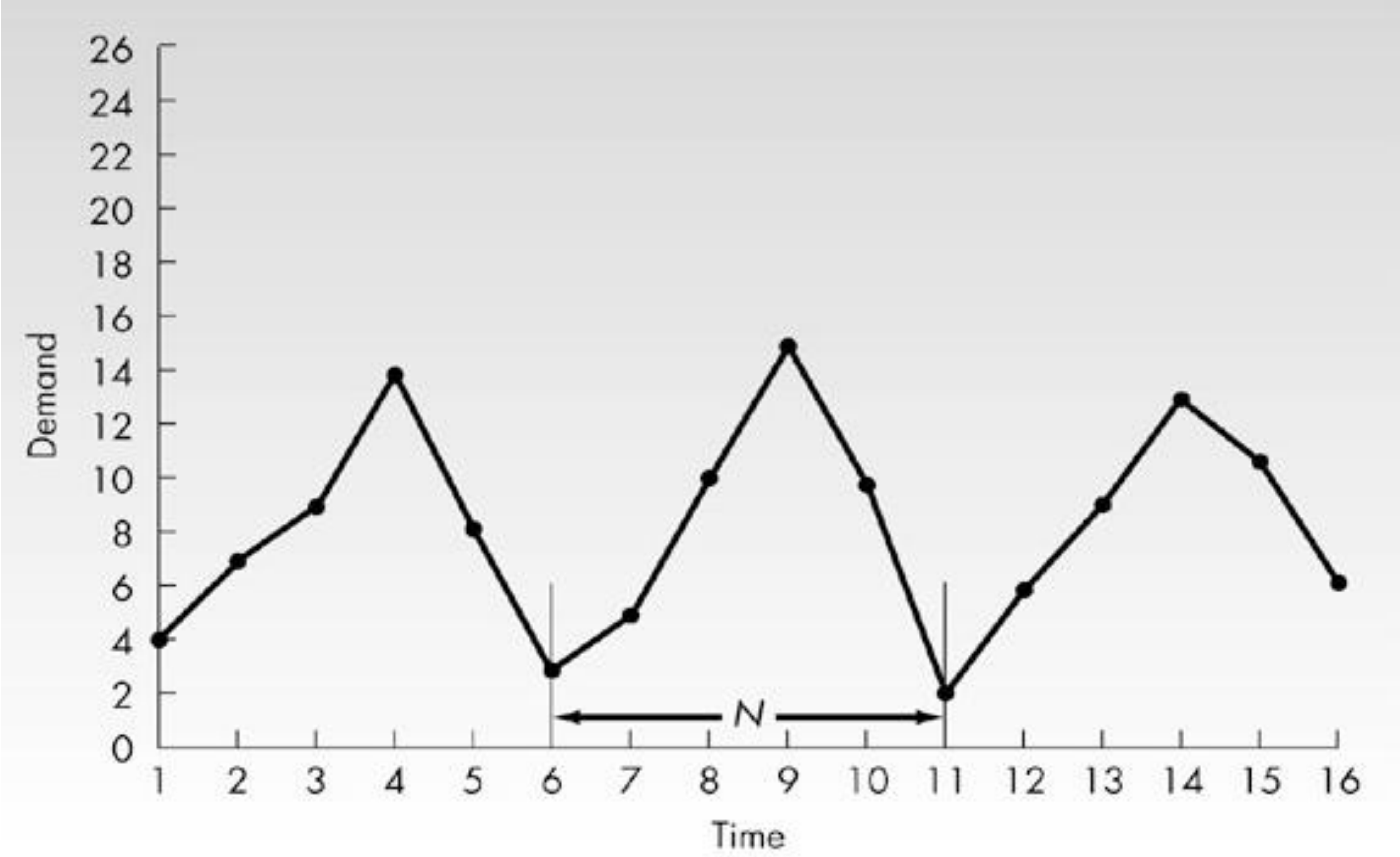
2. *Forecast for the deseasonalized data*

   The resulting series will have no seasonality and may then be predicted using an appropriate method

3. *Incorporate seasonality in a forecast*

   Add seasonality by multiplying by the corresponding seasonal relative to obtain a forecast for the original series.

A SEASONAL DEMAND SERIES



SEASONAL FACTORS FOR STATIONARY SERIES

- Quick and dirty method of estimating seasonal factors
- Compute the sample mean of the entire data set (should be at least several cycles of data)
- Divide each observation by the sample mean: this gives a factor for each observation
- Average the factors for like seasons

→ The resulting n numbers will exactly add to N and correspond to the N seasonal factors

SEASONAL DECOMPOSITION USING CMA

- Slightly more complex, but can be used to predict a seasonal series with or without a trend

  → an example with N=4

(A) Period	(B) Demand	MA(4)		(C) Centered	(B/C) Ratio
1	10			18.81	0.532
2	20			18.81	1.063
			18.25		
3	26			18.50	1.405
			18.75		
4	17	18.25		19.125	0.888
			19.50		
5	12	18.75		20.00	0.600
			20.50		
6	23	19.50		21.125	1.089
			21.75		
7	30	20.50		20.56	1.463
8	22	21.75		20.56	1.070

SEASONAL DECOMPOSITION USING CMA

- The next step is to average the factors for like seasons AND normalize

  $c_1 = 0.558 \;\; c_2 = 1.061 \;\; c_3 = 1.415 \;\; c_4 = 0.966 \qquad\qquad \sum_{t=1}^{4} c_t = 4$
- Then you can deseasonalize demand by dividing each observation by the appropriate factor

Period	Factor	Deseasonalized Demand
1	0.558	17.92
2	1.061	18.85
3	1.415	18.39
4	0.966	17.60
5	0.558	21.50
6	1.061	21.68
7	1.415	21.22
8	0.966	22.77

BUT WHAT ABOUT NEW DATA?

- Same problem prevails as before: updating is ‘expensive’
- As new data becomes available, we must start over to get seasonal factors, trend and intercept estimates
- Is there a method to smooth this seasonalized technique?
- Yes, it is called Winter’s Method or triple exponential smoothing

WINTERS'S METHOD

- This model uses 3 smoothing equations: one for the signal, one for the trend, and one for seasonal factors
- The equations may have different smoothing constants α, β and γ
- The series:

$$S_t = \alpha(D_t/c_{t-N}) + (1-\alpha)(S_{t-1} + G_{t-1})$$

- The trend:

$$G_t = \beta[S_t - S_{t-1}] + (1-\beta)G_{t-1}$$

- The seasonal factors:

$$c_t = \gamma(D_t/S_t) + (1-\gamma)c_{t-N}$$

- $F_{t,t+\tau} = (S_t + \tau * G_t)c_{t+\tau-N}$ τ-step-ahead forecast under the assumption $\tau \leq N$

INITIALIZATION PROCEDURE WINTERS

- We must derive initial estimates of the 3 values:

  S_0, G_0 and c_0
- Deriving initial estimates takes at least two complete cycles of data

We explain the procedure for exactly two cycles. It can be generalized for more cycles.

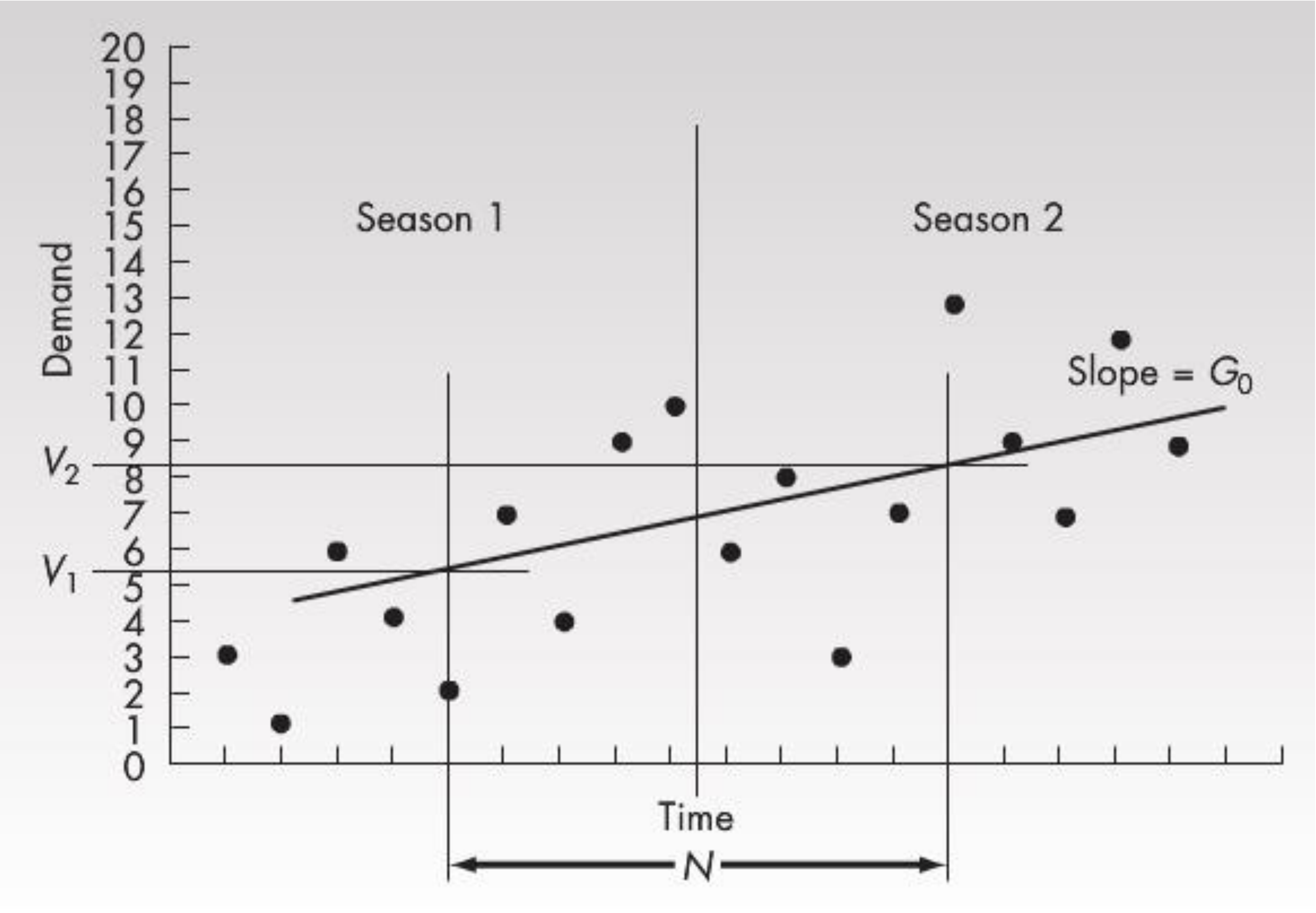
1. Compute sample means for two separate cycles of data (V_1 and V_2)

$$V_1 = \frac{1}{N} \sum_{j=-2N+1}^{-N} D_j$$

$$V_2 = \frac{1}{N} \sum_{j=-N+1}^{0} D_j$$

INITIALIZATION PROCEDURE WINTERS (CONTD)

2. Define $G_0 = (V_2 - V_1)/N$ as the initial slope estimate
3. Set $S_0 = V_2 + G_0[(N-1)/2]$



INITIALIZATION PROCEDURE WINTERS (CONTD)

4. Determine seasonal factors

a) The initial seasonal factors are computed for each period

$$c_t = \frac{D_t}{V_i - [(N+1)/2 - j]G_0} \qquad for - 2N + 1 \leq t \leq 0$$

where i is the cycle and j is the period of the cycle

b) Average the seasonal factors (assuming exactly two cycles of initial data)

$$c_{-N+1} = \frac{c_{-2N+1} + c_{-N+1}}{2}, \dots, c_0 = \frac{c_{-N} + c_0}{2}$$

b) Normalize the seasonal factors

$$c_j = \left[\frac{c_j}{\sum_{i=-N+1}^{0} c_i}\right] \times N \qquad for - N + 1 \leq j \leq 0$$

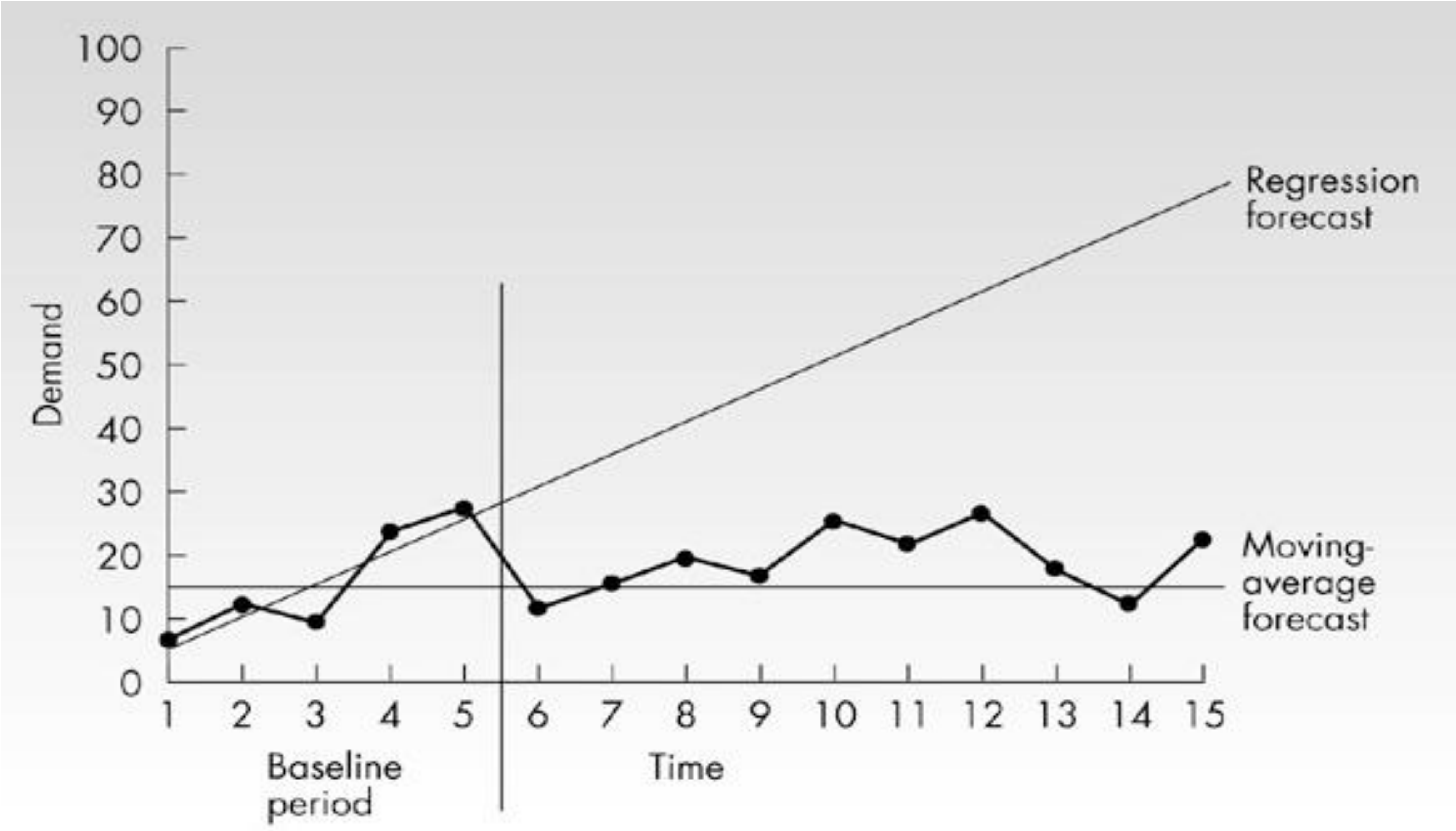
OUTLINE

CONCLUSION

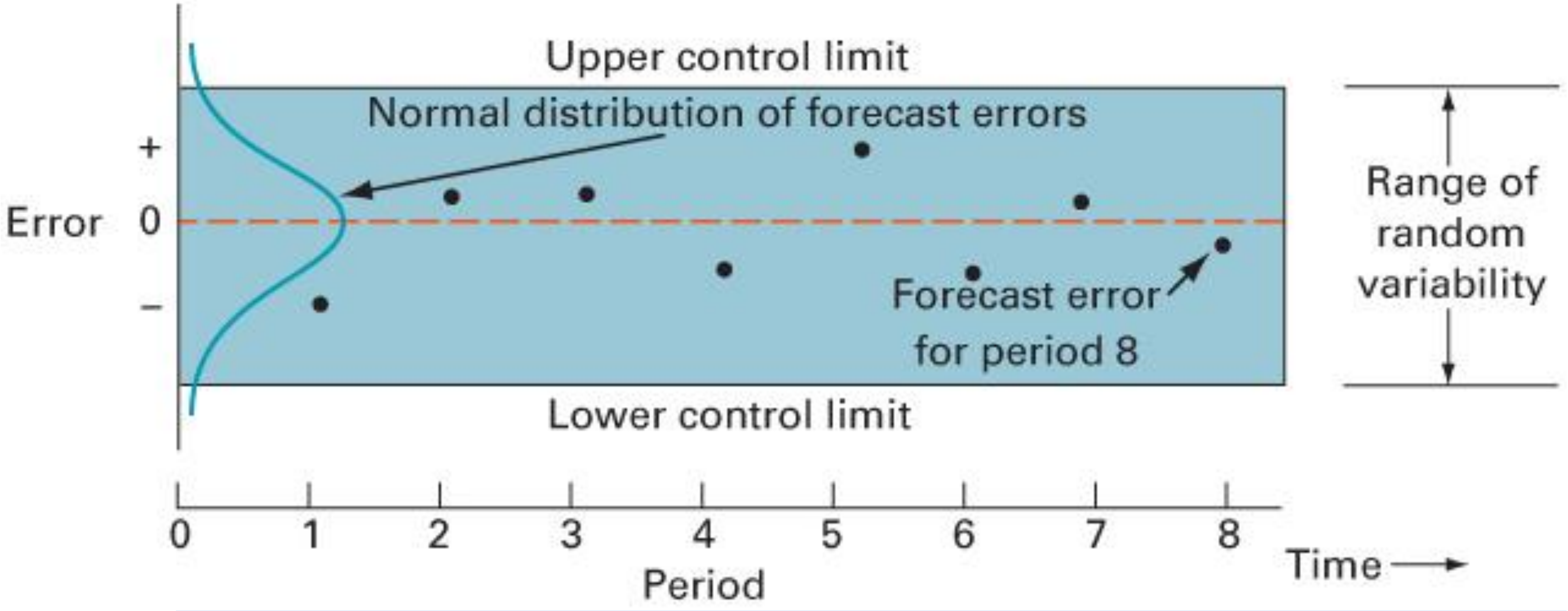
Practical considerations

- Determine the proper model: consider the context and graph historical data to spot patterns
- Overly sophisticated forecasting methods can be problematic, especially for long term forecasting. (Figure on the next slide)
- Tracking signals and control charts may be useful for indicating forecast bias.
- Some evidence exists that averages of forecasts from different methods are more accurate than a single method

DIFFICULTY WITH LONG-TERM FORECASTS



CONTROL CHART CONSTRUCTION



1. Compute the MSE.
2. Estimate of standard deviation of the distribution of errors

$$s = \sqrt{\text{MSE}}$$

3. $\text{UCL}: 0 + z\sqrt{\text{MSE}}$
4. $\text{LCL}: 0 - z\sqrt{\text{MSE}}$

where z = Number of standard deviations from the mean

CONCLUSION

Characteristics of forecasts

- They are usually wrong!
- A good forecast is more than a single number
- Aggregate forecasts are usually more accurate
- Accuracy erodes as we go further into the future.
- Forecasts should not be used to the exclusion of known information

QUESTIONS/REMARKS